Rational Functions(3)

Consider the function f defined over $IR^*$by $f(x) = \frac{x^3 - x + 1}{x^2}$. (C) its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$.

1. Calculate the limits of f at the boundaries of D.

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^3}{x^2} = \lim_{x \to -\infty} x = -\infty$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x = +\infty$$

$$\lim_{x \to 0^-} f(x) = \frac{1}{0^+} = +\infty$$

$$\lim_{x \to 0^+} f(x) = \frac{1}{0^+} = +\infty$$

So $x = 0$ is a vertical asymptote

Consider the function f defined over $IR^*$by $f(x) = \frac{x^3-x+1}{x^2}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

2. Determine a, b, c and d so that $f(x) = ax + b + \frac{cx+d}{x^2}$. Deduce that the line (d) of equation $ax + b$ is an oblique asymptote.

$$ax + b + \frac{cx+d}{x^2} = \frac{(ax+b)x^2+cx+d}{x^2} = \frac{ax^3+bx^2+cx+d}{x^2} = \frac{x^3-x+1}{x^2}$$

By comparing the numerators:

$a = 1 \quad ; \quad b = 0 \quad ; \quad c = -1 \quad ; \quad d = 1$

So $f(x) = x + \frac{-x+1}{x^2}$

$$f(x) - y = x + \frac{-x+1}{x^2} - x = \frac{-x+1}{x^2}$$

$\lim\limits_{x\to\pm\infty} \frac{-x+1}{x^2} = \lim\limits_{x\to\pm\infty} -\frac{x}{x^2} = \lim\limits_{x\to\pm\infty} -\frac{1}{x} = 0$ so (d) is an oblique asymptote.

Consider the function f defined over $IR^*$by $f(x) = \frac{x^3-x+1}{x^2}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

3. Study the relative position between (d) and (C).

$f(x) - y = x + \frac{-x+1}{x^2} - x = \frac{-x+1}{x^2}$; $-x+1=0$; $x=1$

x		0		1	
$f(x)-y$	+	‖	+	0	−
Position	(C) is above (d)	‖	(C) is above (d)	(C) cuts (d) at (1;1)	(C) is below (d)

Consider the function f defined over $IR^*$by $f(x) = \frac{x^3-x+1}{x^2}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

4.Show that $f'(x) = \frac{(x-1)(x^2+x+2)}{x^3}$ and set up the table of variations of f.

$$f'(x) = \frac{(3x^2-1)(x^2)-2x(x^3-x+1)}{(x^2)^2} = \frac{3x^4-x^2-2x^4+2x^2-2x}{x^4} = \frac{x^4+x^2-2x}{x^4} = \frac{x(x^3+x-2)}{x^4}$$

$$= \frac{x^3+x-2}{x^3}$$

$$(x-1)(x^2+x+2) = x^3+x^2+2x-x^2-x-2 = x^3+x-2$$

So $f'(x) = \frac{(x-1)(x^2+x+2)}{x^3}$

$f'(x) = 0$; $x-1=0$ or $x^2+x+2=0$

$x = 1$ $\Delta = 1^2 - 4(1)(2) = -7 < 0$ so no roots

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4.Show that $f'(x) = \frac{(x-1)(x^2+x+2)}{x^3}$ and set up the table of variations of f.

x		0		1	
$x-1$	$-$	||	$-$	0	$+$
x^2+x+2	$+$	||	$+$		$+$
x^3	$-$	||	$+$		$+$
$f'(x)$	$+$	||	$-$	0	$+$
$f(x)$	$-\infty$ ↗ $+\infty$	||	$+\infty$ ↘	$f(1) = 1$	↗ $+\infty$

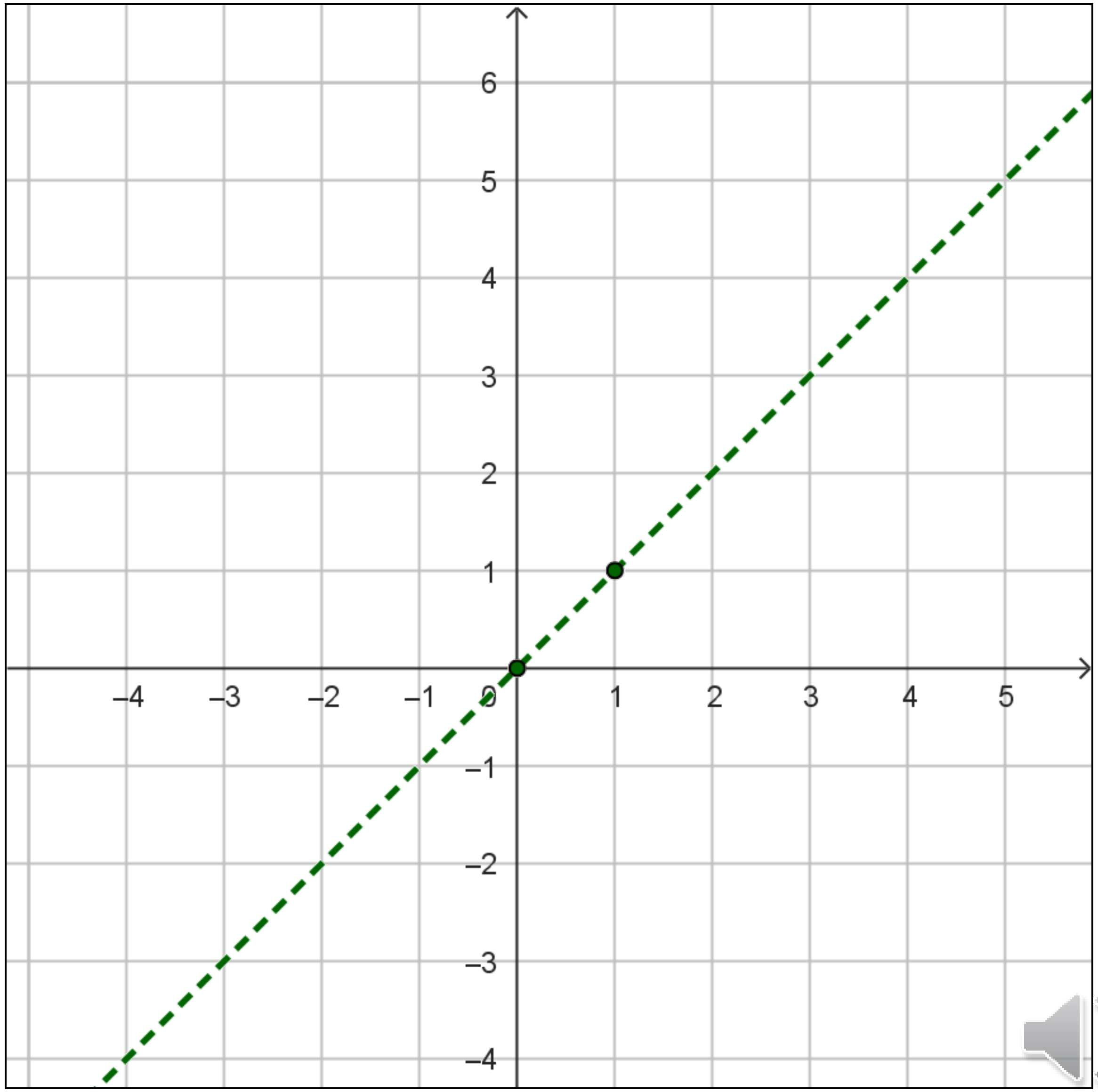
Consider the function f defined over IR^* by $f(x) = \frac{x^3 - x + 1}{x^2}$. (C) its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$.

5. Plot (C) knowing that (C) cuts $(x'x)$ at $x \approx -1.3$

- Start by the asymptotes:

$x = 0$ vertical line which is (y'y)

$y = x$ Oblique line



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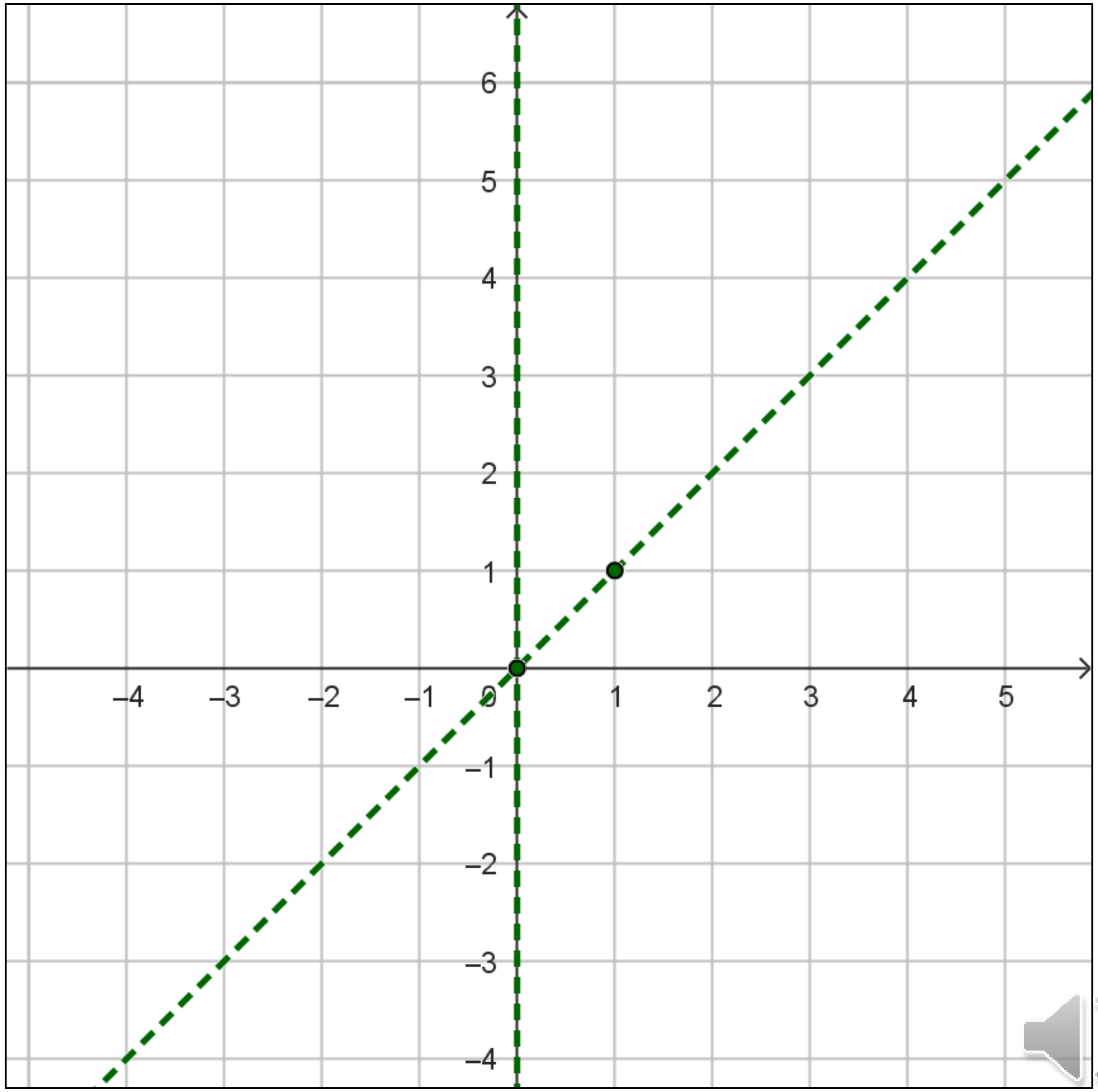

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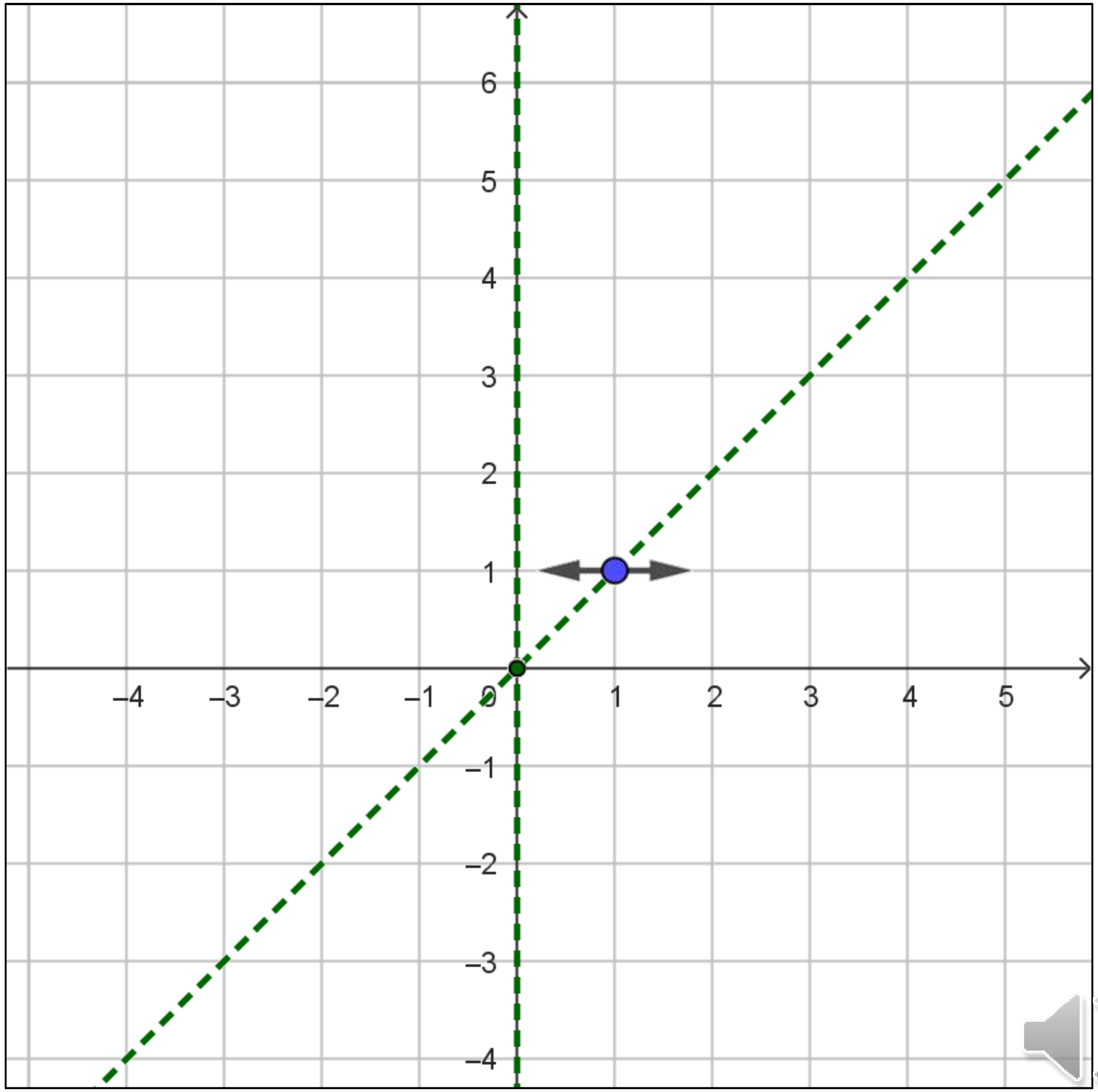
5. Plot (C) knowing that (C) cuts $(x'x)$ at $x \approx -1.3$

- Plot the extrema

x		0		1	
$x-1$	$-$	‖	$-$	0	$+$
x^2+x+2	$+$	‖	$+$		$+$
x^3	$-$	‖	$+$		$+$
$f'(x)$	$+$	‖	$-$	0	$+$
$f(x)$	$-\infty$ ↗ $+\infty$	‖	$+\infty$ ↘	$f(1)=1$	↗ $+\infty$



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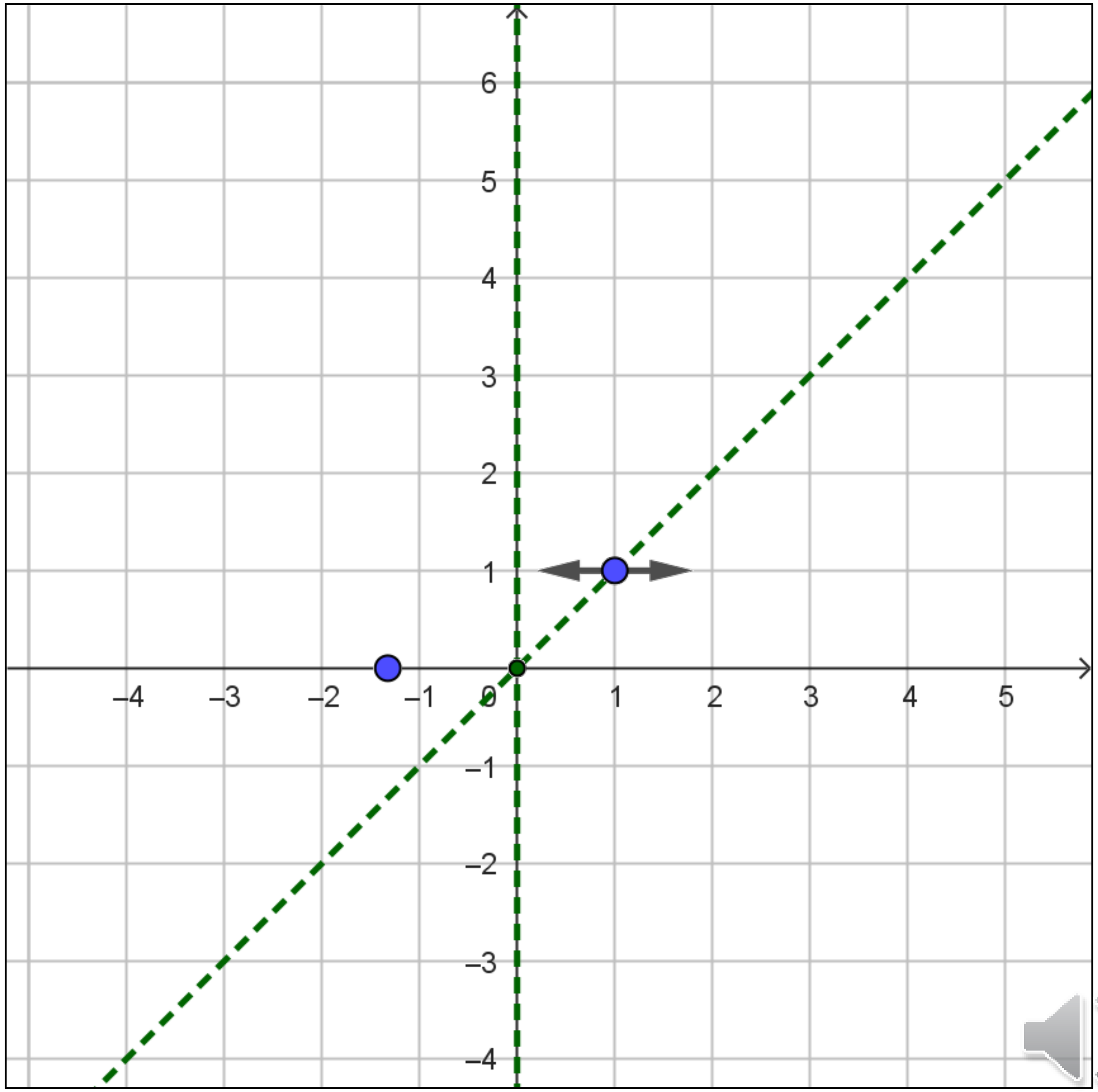
5. Plot (C) knowing that (C) cuts $(x'x)$ at $x \approx -1.3$

➢ Plot the particular points



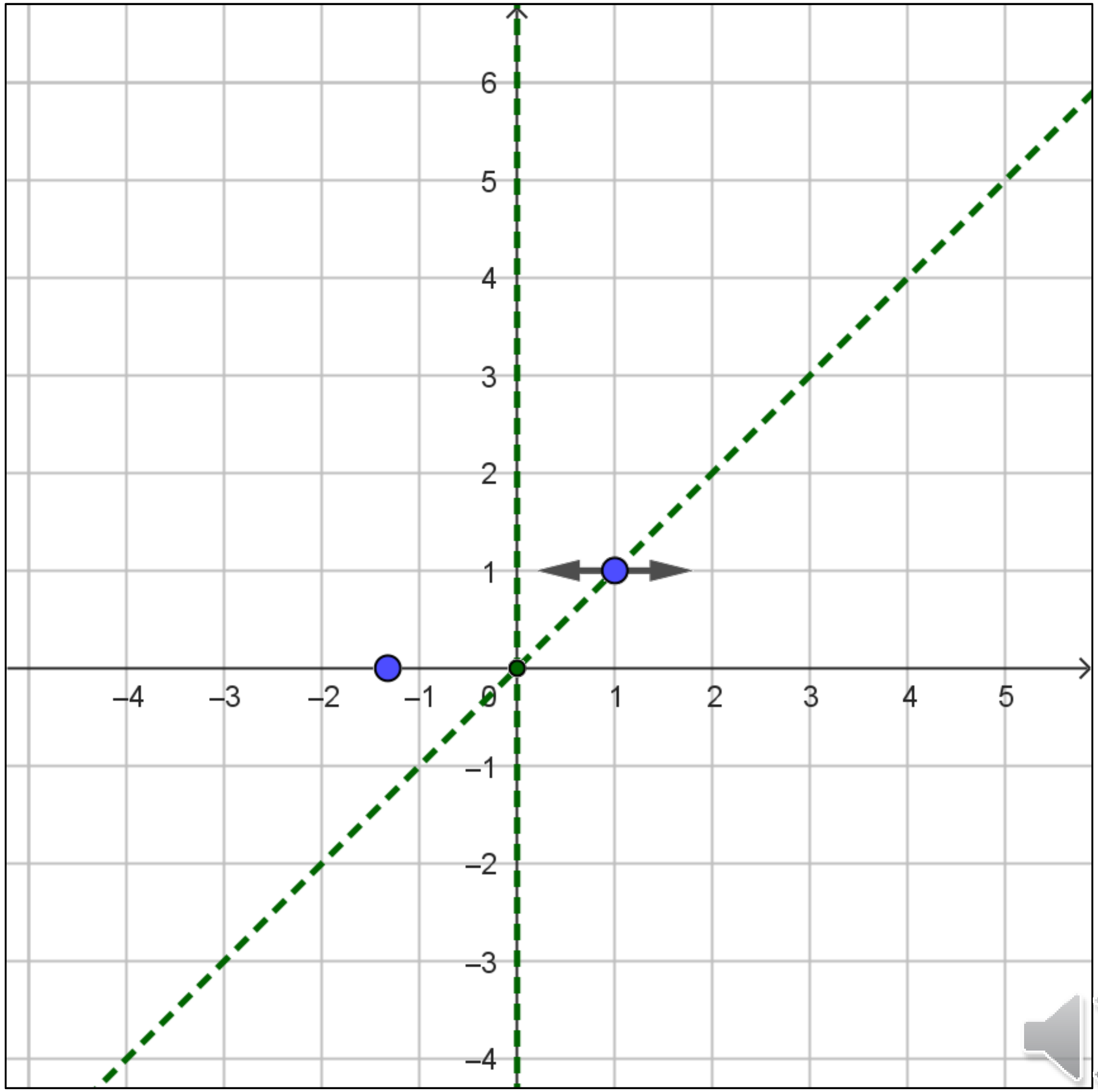
Consider the function f defined over $IR^*$by $f(x) = \frac{x^3-x+1}{x^2}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

5. Plot (C) knowing that (C) cuts $(x'x)$ at $x \approx -1.3$

- Start drawing based on the table of variations

x		0		1	
$x-1$	$-$	‖	$-$	0	$+$
x^2+x+2	$+$	‖	$+$		$+$
x^3	$-$	‖	$+$		$+$
$f'(x)$	$+$	‖	$-$	0	$+$
$f(x)$	-∞ ↗ $+\infty$	‖	$+\infty$ ↘	$f(1)=1$	↗ $+\infty$

O.A.



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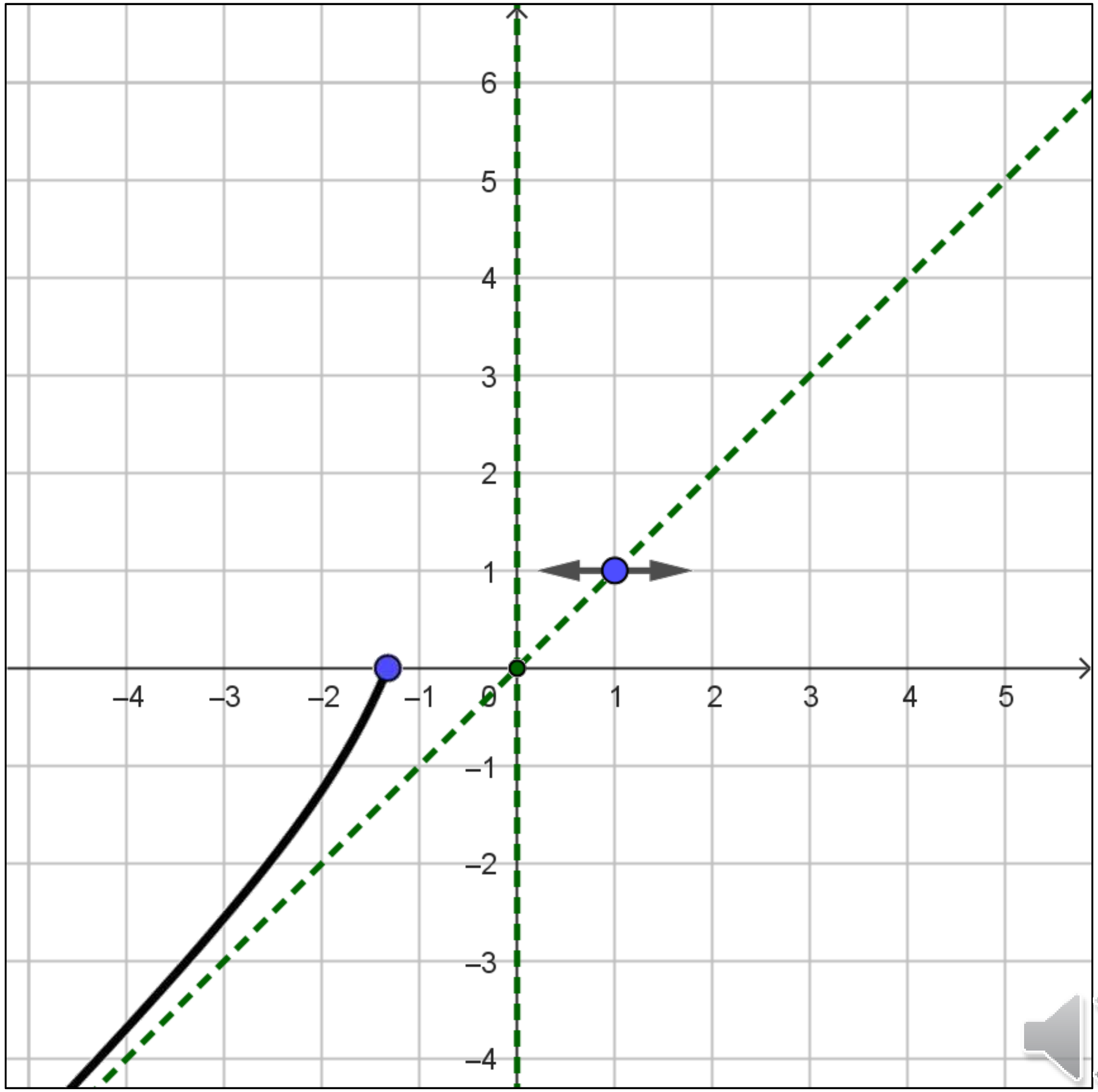
5. Plot (C) knowing that (C) cuts (x'x) at $x \approx -1.3$

➢ Start drawing based on the table of variations

x		0		1	
$f(x) - y$	+	(hatched)	+	0	−
Position	(C) is above (d)		(C) is above (d)		(C) is below (d)

(C) cuts (d) at (1;1)



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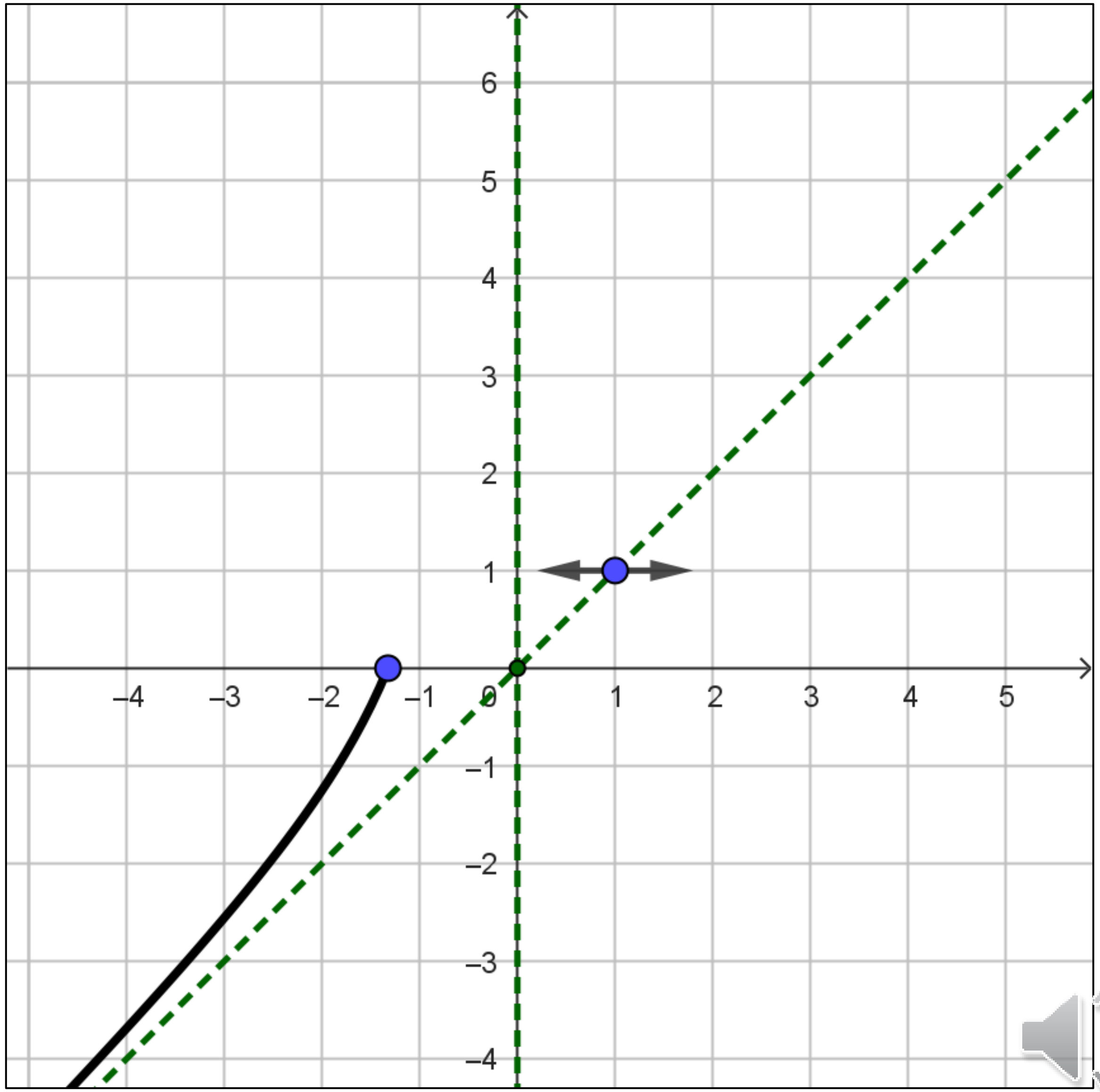
x		0		1	
$x-1$	$-$	||	$-$	0	$+$
x^2+x+2	$+$	||	$+$		$+$
x^3	$-$	||	$+$		$+$
$f'(x)$	$+$	||	$-$	0	$+$
$f(x)$	$-\infty \nearrow +\infty$	||	$+\infty \searrow$	$f(1)=1$	$\nearrow +\infty$

V.A.



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- Start drawing based on the table of variations

x		0		1	
$x-1$	$-$		$-$	0	$+$
x^2+x+2	$+$		$+$		$+$
x^3	$-$		$+$		$+$
$f'(x)$	$+$		$-$	0	$+$
$f(x)$	$-\infty \nearrow +\infty$		$+\infty \searrow$	$f(1)=1$	$\nearrow +\infty$

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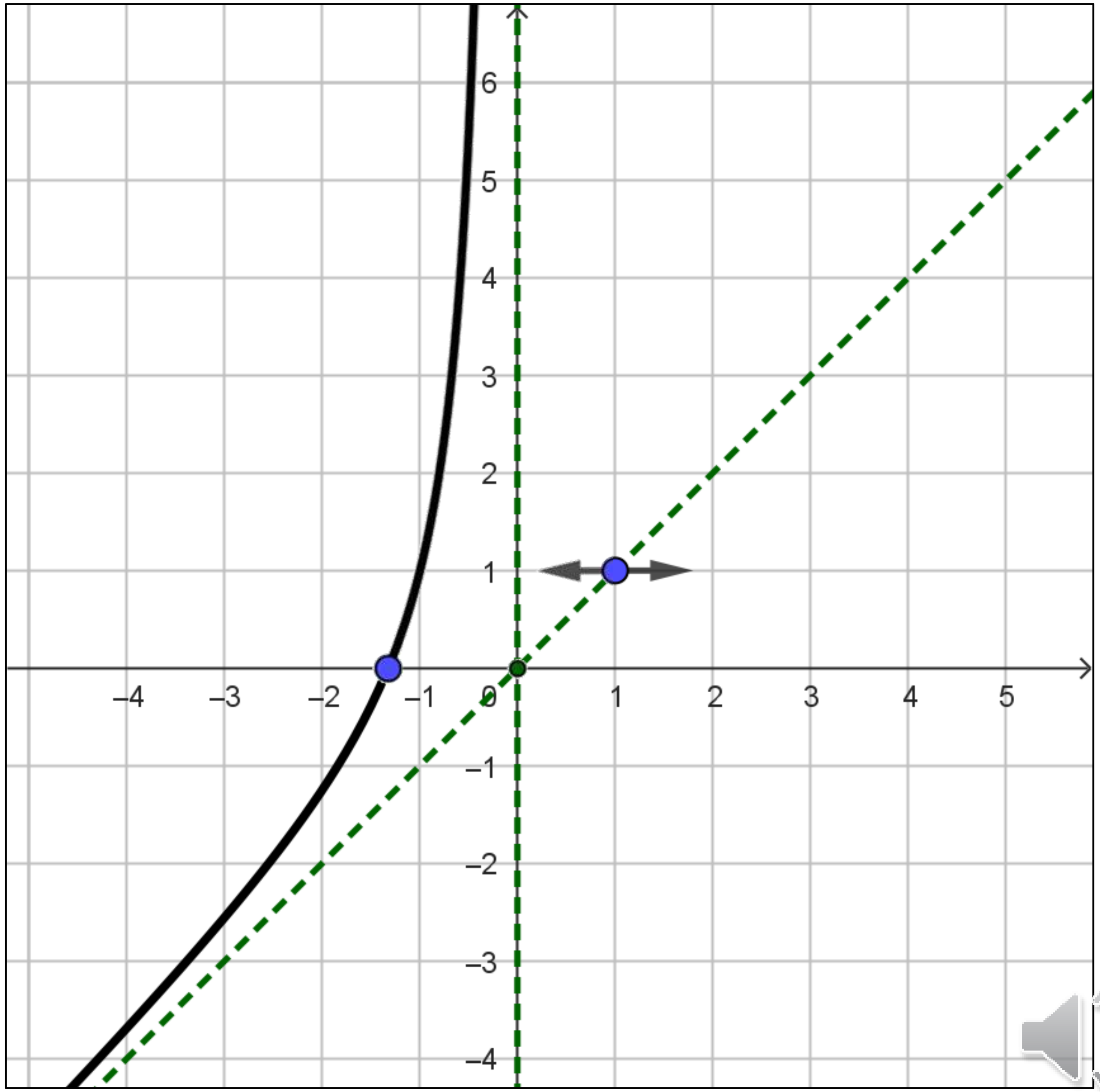

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$x-1$	$-$	‖	$-$	0	$+$
x^2+x+2	$+$	‖	$+$		$+$
x^3	$-$	‖	$+$		$+$
$f'(x)$	$+$	‖	$-$	0	$+$
$f(x)$	$-\infty$ ↗ $+\infty$	‖	$+\infty$ ↘	$f(1)=1$	↗ $+\infty$

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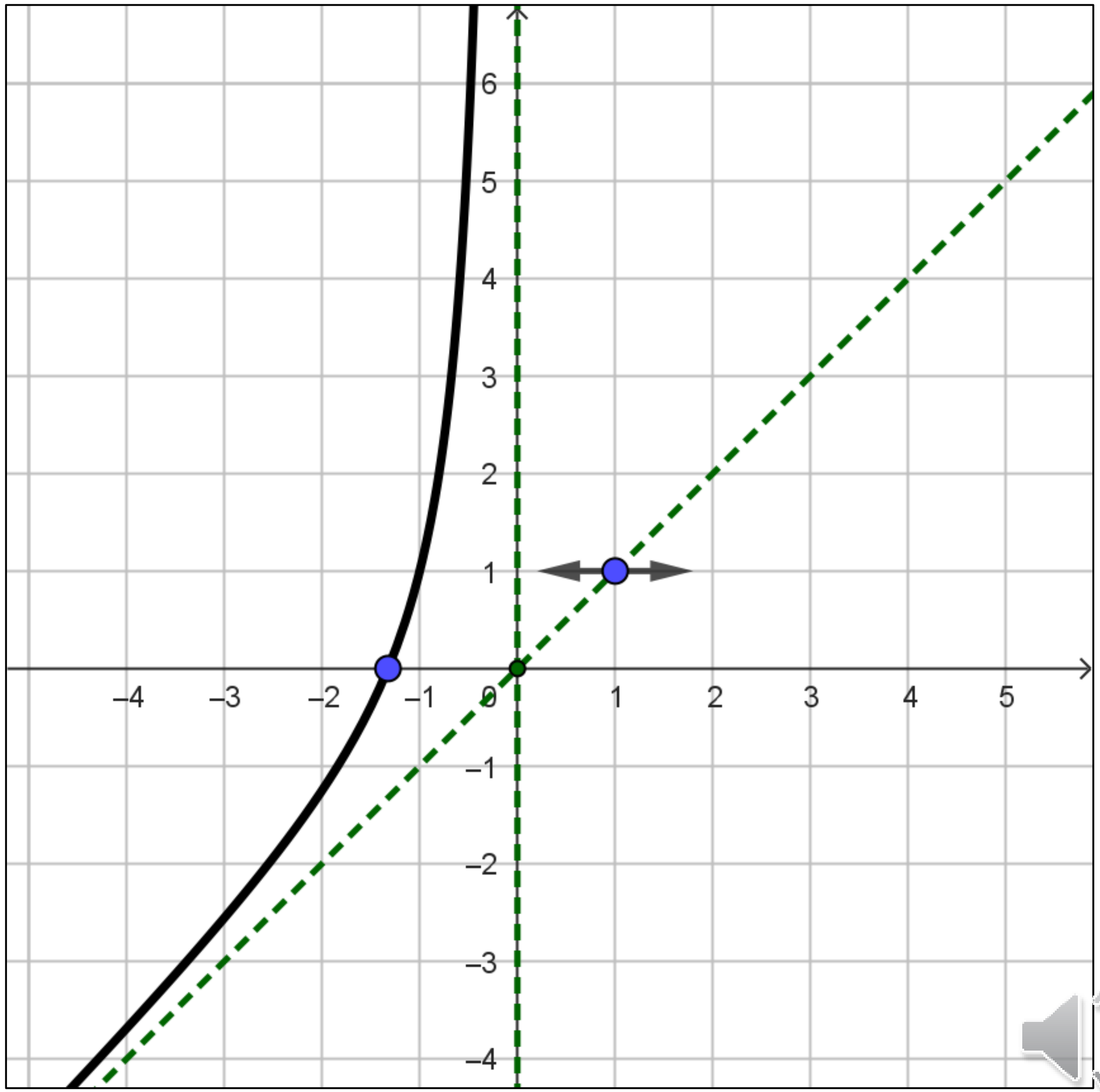

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- Start drawing based on the table of variations

		0		1	
$x - 1$	$-$		$-$	0	$+$
$x^2 + x + 2$	$+$		$+$		$+$
x^3	$-$		$+$		$+$
$f'(x)$	$+$		$-$	0	$+$
$f(x)$	$-\infty \nearrow +\infty$		$+\infty \searrow$	$f(1) = 1$	$\nearrow +\infty$

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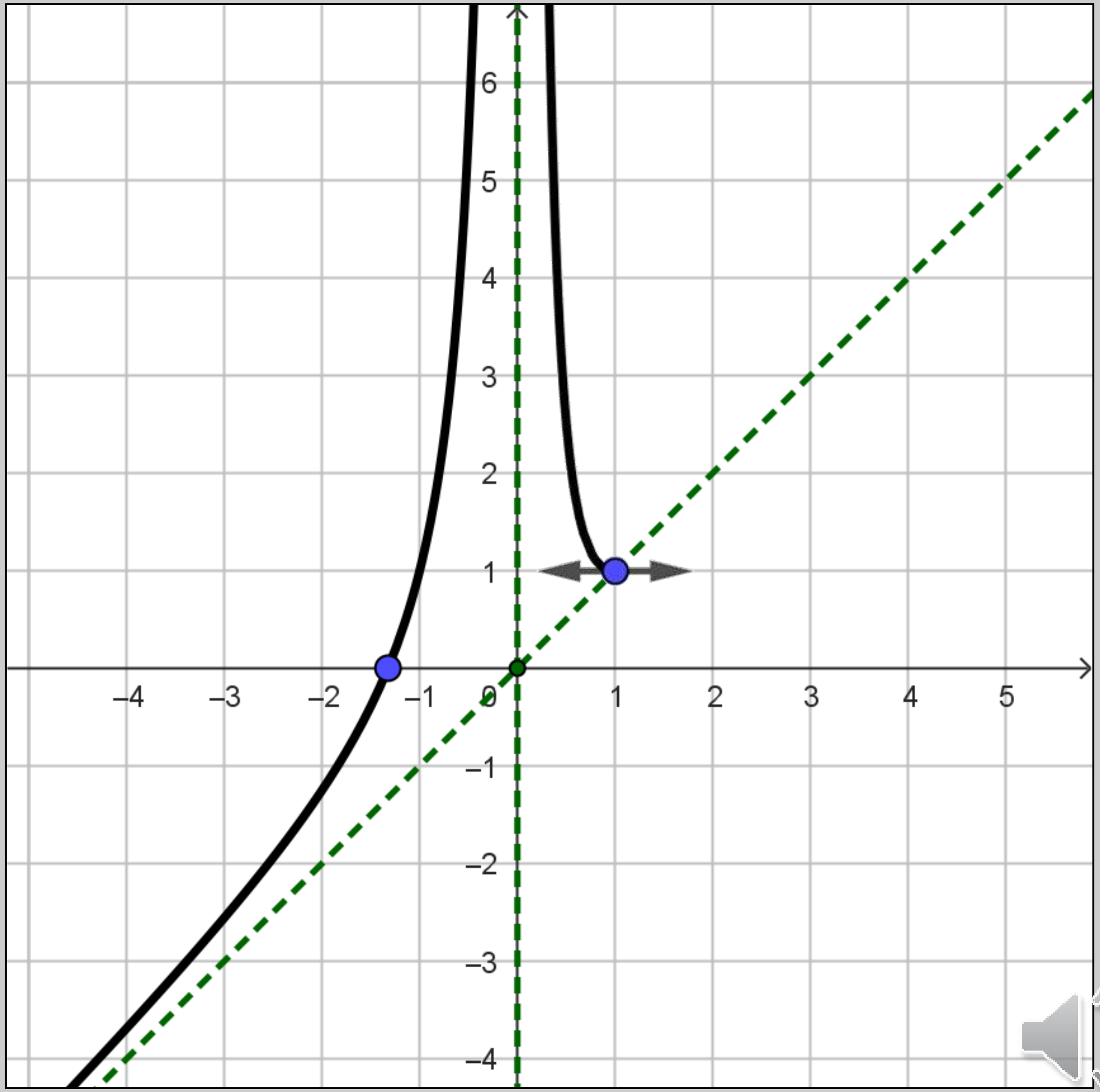

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x		0		1	
$x-1$	$-$	||	$-$	0	$+$
x^2+x+2	$+$	||	$+$		$+$
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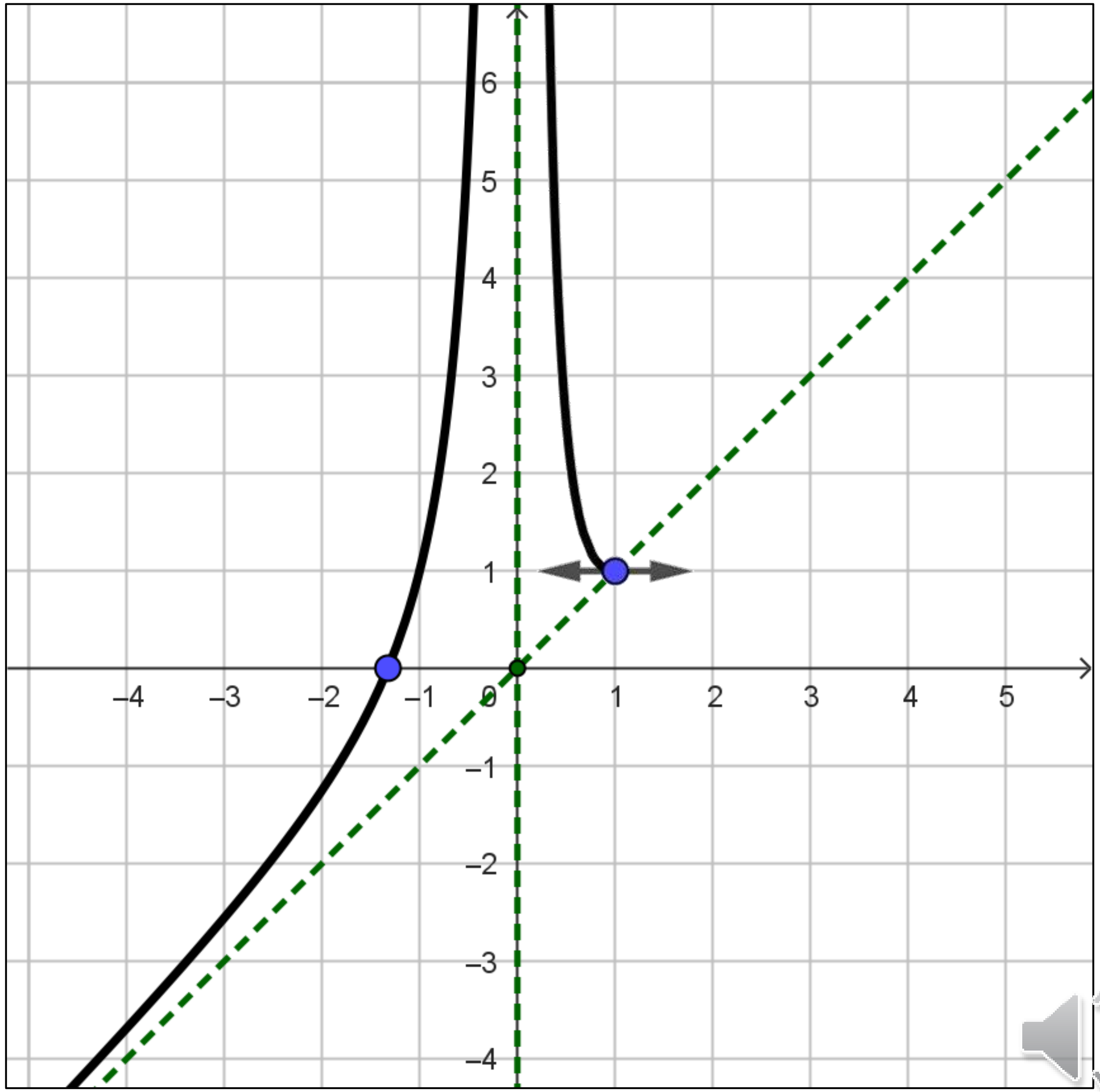

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$f(x) - y$	+	‖	+	0	−
Position	(C) is above (d)	‖	(C) is above (d)		(C) is below (d)

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